Level – 2	Daily Tutorial Sheet – 10

121.(D) $I = \int \frac{x^4-1}{x^2\left(x^4+x^2+1\right)^{1/2}}\,dx = \int \frac{\left(x^2-1\right)\left(x^2+1\right)dx}{x^3\sqrt{x^2+1+\frac{1}{x^2}}} = \int \frac{\left(x+\frac{1}{x}\right)\left(1-\frac{1}{x^2}\right)dx}{\sqrt{\left(x+\frac{1}{x}\right)^2-1}}$

Put $x+\frac{1}{x}=t$ to get $I = \int \frac{t\,dt}{\sqrt{t^2-1}} = \frac{1}{2}\int \frac{2t\,dt}{\sqrt{t^2-1}}$

Put $t^2-1=u$ to get $I = \frac{1}{2}\int \frac{du}{\sqrt{u}} = \sqrt{u}+C = \frac{\sqrt{x^4+x^2+1}}{x}+C$

122.(B) Let $\tan^{-1}\left(\sec x+\cos x\right)=t \Rightarrow \frac{1}{1+\left(\sec x+\cos x\right)^2}\left(\sec x\ \tan x-\sin x\right)dx = dt \Rightarrow \frac{\sin^3 x\,dx}{\cos^4 x+3\cos^2 x+1}=dt$

$\therefore \quad I = \int \frac{dt}{t} = \ell n\left|t\right|+C = \ell n \tan^{-1}\left(\sec x+\cos x\right)+C$

123.(B) $\int \frac{e^{\cos x}\left(x\sin^3 x+\cos x\right)}{\sin^2 x}dx$ [Put $\cos x = t$, we get : $-\sin x\,dx = dt \Rightarrow dx = \frac{-dt}{\sqrt{1-t^2}}$]

$\int e^t\left(\cos^{-1}t-\frac{1}{\sqrt{1-t^2}}+\frac{1}{\sqrt{1-t^2}}+\frac{1}{\left(1-t^2\right)^{3/2}}\right)dt = e^t\left(\cos^{-1}t+\frac{1}{\sqrt{1-t^2}}\right)+C = e^{\cos x}\left(x+\operatorname{cosec} x\right)+C$

124.(B) $I = \int \frac{x^2-1}{x\sqrt{1+x^4}}dx = \int \frac{\left(1-\frac{1}{x^2}\right)}{\sqrt{x^2+\frac{1}{x^2}}}dx$ [put $x+\frac{1}{x}=t \Rightarrow \left(1-\frac{1}{x^2}\right)dx = dt$]

$I = \int \frac{dt}{\sqrt{t^2-\left(\sqrt{2}\right)^2}} = \ell n\left|t+\sqrt{t^2-2}\right|+C = \ell n\left|\left(x+\frac{1}{x}\right)+\sqrt{x^2+\frac{1}{x^2}}\right|+C$

125.(C) $I = \int \frac{\left(\frac{1}{x^2}+1\right)}{\left(\frac{1}{x}-x\right)\sqrt{\frac{1}{x^2}+x^2}}dx = \int \frac{\left(\frac{1}{x^2}+1\right)}{\left(\frac{1}{x}-x\right)\sqrt{\left(x-\frac{1}{x}\right)^2+2}}dx$ [Let $x-\frac{1}{x}=t \Rightarrow \left(1+\frac{1}{x^2}\right)\cdot dx = dt$]

$I = \int \frac{-dt}{t\sqrt{t^2+2}} = \int \frac{-t\,dt}{t^2\sqrt{t^2+2}}$ [Let $\sqrt{t^2+2}=u \Rightarrow t^2 = u^2-2 \Rightarrow 2t\,dt = 2u\,du$]

$I = \int \frac{-u\,du}{\left(u^2-2\right)u} = \int \frac{-du}{u^2-\left(\sqrt{2}\right)^2} = \frac{-1}{2\sqrt{2}}\ell n\left|\frac{u-\sqrt{2}}{u+\sqrt{2}}\right| = \frac{-1}{2\sqrt{2}}\ell n\left(\frac{\left(u-\sqrt{2}\right)^2}{u^2-2}\right)$

$= \frac{-1}{\sqrt{2}}\ell n\left(\frac{\sqrt{t^2+2}-\sqrt{2}}{t}\right) = \frac{-1}{\sqrt{2}}\ell n\left(\frac{\sqrt{\left(x-\frac{1}{x}\right)^2+2}-\sqrt{2}}{\left(x-\frac{1}{x}\right)}\right)$

126.(D) $I = \int \frac{\frac{1}{x^2}}{\left(1+\frac{1}{x^4}\right)\sqrt{\left(\sqrt{1+\frac{1}{x^4}}\right)-1}}dx$ [Put $\sqrt{\left(1+\frac{1}{x^4}\right)}-1=t \Rightarrow 1+\frac{1}{x^4}=\left(1+t^2\right)^2 \Rightarrow \frac{-4}{x^5}dx = 2\left(1+t^2\right)2t\,dt$]

$\therefore \quad I = \int \frac{-\left(1+t^2\right)t\,dt}{\left(1+t^2\right)^2 t} = -\int \frac{1}{1+t^2}dt = -\tan^{-1}(t)+C = -\tan^{-1}\sqrt{\left(\sqrt{1+\frac{1}{x^4}}\right)-1}+C$

127.(D) $I = \int \frac{dx}{\left((x+2)^2+1\right)^2}$ [$x+2=\tan\theta \Rightarrow dx = \sec^2\theta$]

$= \int \frac{\sec^2\theta}{\left(1+\tan^2\theta\right)^2}d\theta = \int \cos^2\theta\, d\theta = \int \frac{1+\cos 2\theta}{2}d\theta = \frac{1}{2}\left[\theta + \frac{\sin 2\theta}{2}\right]+c = \frac{1}{2}\left[\theta+\sin\theta\cos\theta\right]+C$

$= \frac{1}{2}\left[\tan^{-1}(x+2)+\frac{x+2}{x^2+4x+5}\right]+C$

128.(B) $\int \frac{\left(2\cos^2 x - 1 - 3\right)\cot x}{\sin^4 x\sqrt{4\cot^2 x-1}}dx$; $\int\left(\frac{2\cos^2 x\,.\cot x}{\sin^4 x\sqrt{4\cot^2 x-1}}-\frac{4\cot x}{\sin^4 x\sqrt{4\cot^2 x-1}}\right)dx$

$\int \frac{2\cot^2 x\,\cot x\,\mathrm{cosec}^2 x}{\sqrt{4\cot^2 x-1}}\,dx - \int \frac{4\cot x\,\mathrm{cosec}^2 x\,\mathrm{cosec}^2 x\,dx}{\sqrt{4\cot^2 x-1}}dx$ (Let $4\cot^2 x - 1 = t^2$) Integrate to get the answer

129.(A) $I = \int \frac{5x^8+7x^6}{x^{14}\left[\frac{1}{x^5}+\frac{1}{x^7}+2\right]^2}\,dx = \int \frac{\left[\frac{5}{x^6}+\frac{7}{x^8}\right]}{\left[\frac{1}{x^5}+\frac{1}{x^7}+2\right]}\,dx$ [Put $\frac{1}{x^5}+\frac{1}{x^7}+2=t \Rightarrow -\left[\frac{5}{x^6}+\frac{7}{x^8}\right]dx = dt$]

$I = \int -\frac{1}{t^2}dt = \frac{1}{t}+C \quad \Rightarrow \quad \frac{x^7}{2x^7+x^2+1}+C$

130.(B) $\sin(2x+\alpha)+\sin\alpha = 2\sin(x+\alpha)\cos x = 2\cos x\left[\sin x\,\cos\alpha+\cos x\,\sin\alpha\right] = 2\cos^2 x\left[\tan x\,\cos\alpha+\sin\alpha\right]$

Now $\quad I = \int \frac{\sec x\,dx}{\sqrt{2}\cos x\left[\tan x\,\cos\alpha+\sin\alpha\right]^{1/2}} = \frac{1}{\sqrt{2}}\int \frac{\sec^2 x\,dx}{\left[\tan x\,\cos\alpha+\sin\alpha\right]^{1/2}}$

[Put $\tan x\,\cos\alpha+\sin\alpha = t \Rightarrow \sec^2 x\,\cos\alpha\,dx = dt$]

$\therefore \quad I = \frac{1}{\sqrt{2}\cos\alpha}\int\frac{dt}{\sqrt{t}} = \frac{1}{\sqrt{2}\cos\alpha}\,.\,2\sqrt{t}+C$

$= \sqrt{2}\left[\frac{\tan x\,\cos\alpha+\sin\alpha}{\cos^2\alpha}\right]^{1/2} = \sqrt{2}\left[\tan x\,\sec\alpha+\sec\alpha\,\tan\alpha\right]^{1/2} = \sqrt{2}\left[(\tan x+\tan\alpha)\sec\alpha\right]^{1/2}+C$